ごうかく!

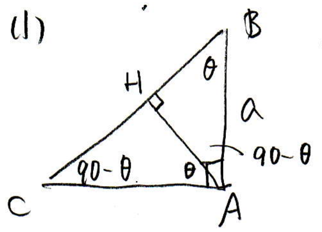
極限24

直角三角形 ABC において, $\angle A = \frac{\pi}{2}$, AB$= a$(一定), 頂点 A から斜辺 BC に引いた垂線の足を H とし, $\angle B = \theta$ とするとき, 次の極限値を求めよ。

(1) $\displaystyle\lim_{\theta\to 0}\frac{\mathrm{CH}}{\theta^2}$ (2) $\displaystyle\lim_{\theta\to 0}\frac{\mathrm{AC}-\mathrm{AH}}{\theta^3}$

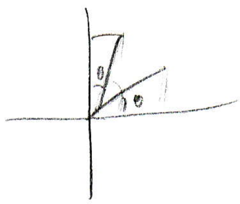
〔早稲田〕

(1)



$\frac{a}{CB} = \sin(90-\theta) = \cos\theta$

$\therefore\ CB = \frac{a}{\cos\theta}$



$\frac{BH}{a} = \sin(90-\theta) = \cos\theta$

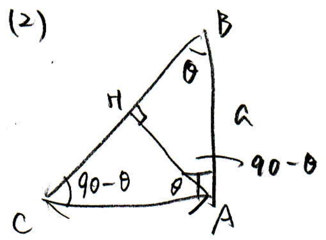
$BH = a\cos\theta$

$\therefore\ CH = CB - BH = \frac{a}{\cos\theta} - a\cos\theta$

$$\lim_{\theta\to 0}\frac{\frac{a}{\cos\theta} - a\cos\theta}{\theta^2} = \lim_{\theta\to 0}\frac{a(1-\cos^2\theta)}{\theta^2\cos\theta} = \lim_{\theta\to 0}\frac{\sin^2\theta}{\theta^2}\cdot\frac{a}{\cos\theta}$$

$= \underline{a}$

(2)



$AH = a\cos(90-\theta)$

$= a\sin\theta$

$\frac{a}{AC} = \tan(90-\theta) = \frac{1}{\tan\theta}$

$AC = a\tan\theta$

$$\frac{a\tan\theta - a\sin\theta}{\theta^3} = \frac{a\sin\theta - a\sin\theta\cos\theta}{\theta^3\cos\theta} = \frac{a\sin\theta(1-\cos\theta)}{\theta^3\cos\theta}$$

$$= \frac{a\sin\theta\cdot(1-\cos^2\theta)}{\theta^3\cos\theta(1+\cos\theta)} = \frac{a\sin^3\theta}{\theta^3}\cdot\frac{1}{\cos\theta(1+\cos\theta)}$$

$$\therefore\ \lim_{\theta\to 0}\frac{a\sin^3\theta}{\theta^3}\,\frac{1}{\cos\theta(1+\cos\theta)} = a\cdot\frac{1}{2} = \underline{\frac{a}{2}}$$