



次の極限を求めよ。

$$\lim_{n \rightarrow \infty} \frac{1}{n} \left\{ \left(a + \frac{1}{n}\right)^2 + \left(a + \frac{2}{n}\right)^2 + \left(a + \frac{3}{n}\right)^2 + \cdots + \left(a + \frac{n}{n}\right)^2 \right\}$$

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$$S_n = \frac{1}{n} \left(a^2 + \frac{2a}{n} + \frac{1}{n^2} + a^2 + \frac{4a}{n} + \frac{4}{n^2} + \cdots + a^2 + 2a + 1 \right)$$

$$= \frac{1}{n} \left\{ na^2 + \frac{a}{n} (2+4+6+\cdots+2n) + \frac{1}{n^2} (1+4+9+\cdots+n^2) \right\}$$

$$= \frac{1}{n} \left\{ na^2 + \frac{a}{n} \cdot 2 \cdot \frac{1}{2} n(n+1) + \frac{1}{n^2} \cdot \frac{1}{6} n(n+1)(2n+1) \right\}$$

$$= \frac{1}{n} \left\{ na^2 + a(n+1) + \frac{(n+1)(2n+1)}{6n} \right\}$$

$$= a^2 + a \left(1 + \frac{1}{n}\right) + \frac{\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right)}{6}$$

∴

$$\lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \left\{ a^2 + a \left(1 + \frac{1}{n}\right) + \frac{\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right)}{6} \right\}$$

$$= \underline{a^2 + a + \frac{1}{3}}$$