3C積分65

次の定積分を求めよ。

(1) $\displaystyle\int_0^{\frac{\pi}{2}} x\sin x\,dx$　　(2) $\displaystyle\int_0^{\frac{\pi}{2}} x\sin^2 x\,dx$

(3) $\displaystyle\int_0^{\frac{\pi}{2}} e^x\sin x\,dx$　　(4) $\displaystyle\int_0^{\frac{\pi}{4}} x\tan^2 x\,x\,dx$　× 訂正

〔基本問題〕

(1)

与式 $= \Big[-x\cos x\Big]_0^{\frac{\pi}{2}} + \displaystyle\int_0^{\frac{\pi}{2}}\cos x\,dx$

$= 0 + \Big[\sin x\Big]_0^{\frac{\pi}{2}} = \underline{1}$ ″

(2)

与式 $= \dfrac{1}{2}\displaystyle\int_0^{\frac{\pi}{2}} x(1-\cos 2x)\,dx = \dfrac{1}{2}\int_0^{\frac{\pi}{2}} x\,dx - \dfrac{1}{2}\int_0^{\frac{\pi}{2}} x\cos 2x\,dx$

$= \dfrac{1}{2}\Big[\dfrac{x^2}{2}\Big]_0^{\frac{\pi}{2}} - \dfrac{1}{2}\left(\Big[\dfrac{x\sin 2x}{2}\Big]_0^{\frac{\pi}{2}} - \dfrac{1}{2}\displaystyle\int_0^{\frac{\pi}{2}}\sin 2x\,dx\right)$

$= \dfrac{\pi^2}{16} + \dfrac{1}{4}\displaystyle\int_0^{\frac{\pi}{2}}\sin 2x\,dx = \dfrac{\pi^2}{16} + \dfrac{1}{4}\Big[-\dfrac{\cos 2x}{2}\Big]_0^{\frac{\pi}{2}}$

$= \dfrac{\pi^2}{16} + \dfrac{1}{4}\left(\dfrac{1}{2}+\dfrac{1}{2}\right)$　∴ $\underline{\dfrac{\pi^2}{16} + \dfrac{1}{4}}$ ″

(3)

与式 $= -\Big[\cos x\cdot e^x\Big]_0^{\frac{\pi}{2}} + \displaystyle\int_0^{\frac{\pi}{2}} e^x\cos x\,dx$

$= 1 + \Big[\sin x\cdot e^x\Big]_0^{\frac{\pi}{2}} - \displaystyle\int_0^{\frac{\pi}{2}} e^x\sin x\,dx = \int_0^{\frac{\pi}{2}} e^x\sin x\,dx$ (与式)

∴ $1 + e^{\frac{\pi}{2}} = 2\displaystyle\int_0^{\frac{\pi}{2}} e^x\sin x\,dx$　∴ $\displaystyle\int_0^{\frac{\pi}{2}} e^x\sin x\,dx = \underline{\dfrac{1+e^{\frac{\pi}{2}}}{2}}$ ″

(4)

与式 $= \displaystyle\int_0^{\frac{\pi}{4}} x\left(\dfrac{1}{\cos^2 x}-1\right)dx = \Big[x\tan x\Big]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}}\dfrac{\sin x}{\cos x}\,dx - \Big[\dfrac{x^2}{2}\Big]_0^{\frac{\pi}{4}}$

$= \dfrac{\pi}{4} + \Big[\log|\cos x|\Big]_0^{\frac{\pi}{4}} - \dfrac{\pi^2}{32}$

$= \dfrac{\pi}{4} + \log\dfrac{1}{\sqrt{2}} - \dfrac{\pi^2}{32}$　∴ $\underline{-\dfrac{1}{2}\log 2 - \dfrac{\pi^2}{32} + \dfrac{\pi}{4}}$ ″