

次の和を求めなさい。

$$(1) S = \frac{1}{2 \cdot 5} + \frac{1}{5 \cdot 8} + \frac{1}{8 \cdot 11} + \dots + \frac{1}{(3n-1)(3n+2)}$$

$$(2) S = 1 \cdot 1 + 3 \cdot 2 + 5 \cdot 2^2 + \dots + (2n-1) \cdot 2^{n-1}$$

$$\begin{aligned} (1) \quad S &= \frac{1}{3} \left\{ \frac{1}{2} - \frac{1}{5} \right\} + \frac{1}{3} \left\{ \frac{1}{5} - \frac{1}{8} \right\} + \frac{1}{3} \left\{ \frac{1}{8} - \frac{1}{11} \right\} \\ &\quad + \dots + \frac{1}{3} \left\{ \frac{1}{3n-1} - \frac{1}{3n+2} \right\} \\ &= \frac{1}{3} \cdot \frac{1}{2} - \frac{1}{3(3n+2)} \\ &= \frac{1}{6} - \frac{1}{3(3n+2)} \\ &= \frac{3n+2-2}{6(3n+2)} \\ &= \frac{n}{2(3n+2)} \end{aligned}$$

$$(2) \quad 2S = 1 \cdot 2 + 3 \cdot 2^2 + 5 \cdot 2^3 + \dots + (2n-1) \cdot 2^n$$

$$S = 1 \cdot 1 + 3 \cdot 2 + 5 \cdot 2^2 + 7 \cdot 2^3 + \dots + (2n-1) \cdot 2^{n-1}$$

$$\rightarrow 2S = \quad 1 \cdot 2 + 3 \cdot 2^2 + 5 \cdot 2^3 + \dots + (2n-3) \cdot 2^{n-1} + (2n-1) \cdot 2^n$$

$$-S = 1 \cdot 1 + 2^2 + 2^3 + 2^4 + \dots + 2^n - (2n-1) \cdot 2^n$$

$$-S = 1 + 2(2 + 2^2 + 2^3 + \dots + 2^{n-1}) - (2n-1) \cdot 2^n$$

$$= 1 + 2 \cdot \sum_{k=1}^n 2^{k-1} - (2n-1) \cdot 2^n$$

$$= 1 + 2 \cdot \frac{2(2^{n-1}-1)}{2-1} - (2n-1) \cdot 2^n$$

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$$= 1 + 2 \cdot 2^n - 4 - 2n \cdot 2^n + 2^n$$

$$= 2^n(3-2n) - 3 \quad \text{よって } S = (2n-3) \cdot 2^n + 3$$