無限級数2

次の無限級数の和を求めよ。

(1) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{2^{n-1}}+\frac{3}{5^{n-1}}\right)$　　(2) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2^{n+1}}{4^{n}}+\frac{6}{3^{n}}\right)$

(3) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{3^{n}}+\frac{1}{5^{n}}\right)$　　(4) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{12}{3^{n+1}}+\frac{5^{n+1}}{10^{n}}\right)$

〔練習問題〕

(1)

$$\frac{1}{1-\frac{1}{2}}+\frac{3}{1-\frac{1}{5}}=2+\frac{15}{4}=\underline{\frac{23}{4}}$$

(2)

$\dfrac{2^{n+1}}{4^n}=\dfrac{4}{4}\left(\dfrac{2^{n-1}}{4^{n-1}}\right)=1\cdot\left(\dfrac{1}{2}\right)^{n-1}$ より $\displaystyle\sum_{n=1}^{\infty}\frac{2^{n+1}}{4^n}=\frac{1}{1-\frac{1}{2}}=2$

$\dfrac{6}{3^n}=\dfrac{6}{3}\left(\dfrac{1}{3^{n-1}}\right)=2\cdot\left(\dfrac{1}{3^{n-1}}\right)$ より $\displaystyle\sum_{n=1}^{\infty}\frac{6}{3^n}=\frac{2}{1-\frac{1}{3}}=3$

$\therefore\ \underline{2+3=5}$

(3)

$\dfrac{2}{3^n}=\dfrac{2}{3}\left(\dfrac{1}{3^{n-1}}\right)$ より $\displaystyle\sum_{n=1}^{\infty}\frac{2}{3^n}=\frac{\frac{2}{3}}{1-\frac{1}{3}}=1$

$\dfrac{1}{5^n}=\dfrac{1}{5}\left(\dfrac{1}{5^{n-1}}\right)$ より $\displaystyle\sum_{n=1}^{\infty}\frac{1}{5^n}=\frac{\frac{1}{5}}{1-\frac{1}{5}}=\frac{1}{4}$

$\therefore\ 1+\dfrac{1}{4}=\underline{\dfrac{5}{4}}$

(4)

$\dfrac{12}{3^{n+1}}=\dfrac{12}{9}\left(\dfrac{1}{3^{n-1}}\right)=\dfrac{4}{3}\left(\dfrac{1}{3^{n-1}}\right)$ より $\displaystyle\sum_{n=1}^{\infty}\frac{12}{3^{n+1}}=\frac{\frac{4}{3}}{1-\frac{1}{3}}=2$

$\dfrac{5^{n+1}}{10^n}=\dfrac{25}{10}\left(\dfrac{5}{10}\right)^{n-1}=\dfrac{5}{2}\left(\dfrac{1}{2}\right)^{n-1}$ より $\displaystyle\sum_{n=1}^{\infty}\frac{5^{n+1}}{10^n}=\frac{\frac{5}{2}}{1-\frac{1}{2}}=5$

$\therefore\ 2+5=\underline{7}$