極限 その 1/2

次の極限値を求めよ。

(1) $\displaystyle \lim_{\substack{\cancel{n\to\infty} \\ x \quad 0}} \frac{\sin(\sqrt{2}x^2)}{\sin^2(\sqrt{2}x)}$ 〔東海大〕

$$\lim_{x\to 0} \frac{\sin(\sqrt{2}x^2)}{\sqrt{2}x^2}\cdot\left(\frac{\sqrt{2}x}{\sin(\sqrt{2}x)}\right)^2\cdot\frac{1}{\sqrt{2}} = \underline{\frac{1}{\sqrt{2}}}$$

$$1 - x^2 - \left(1 - x^2 + \frac{x^4}{4}\right)$$

(2) $\displaystyle \lim_{\substack{\cancel{n\to\infty} \\ x \quad 0}} \frac{\sqrt{1-x^2}-\left(1-\dfrac{x^2}{2}\right)}{\sin^4 x}$ 〔山梨医大〕

$$\lim_{x\to 0} \frac{-\dfrac{x^4}{4}}{\left\{\sqrt{1-x^2}+\left(1-\dfrac{x^2}{2}\right)\right\}\sin^4 x}$$

$$= \lim_{x\to 0} -\frac{1}{4\left\{\sqrt{1-x^2}+\left(1-\dfrac{x^2}{2}\right)\right\}}\cdot\frac{x^4}{\sin^4 x} = \underline{-\frac{1}{8}}$$

(3) $\displaystyle\lim_{\substack{\cancel{x\to\infty} \\ t\to 2\pi}} \frac{\sin t}{t^2 - 4\pi^2}$ 〔東京電機大〕

$t+p=2\pi$ として $p\to 0$ に近づくことを考える

$t=2\pi-p$ となるので

$$\frac{\sin(2\pi-p)}{(2\pi-p)^2-4\pi^2} = -\frac{\sin p}{(2\pi-p+2\pi)(\cancel{2\pi}-p-\cancel{2\pi})}$$

$$= -\frac{\sin p}{p(p-4\pi)}$$

$$\therefore \lim_{p\to 0} -\frac{\sin p}{p(p-4\pi)} = \lim_{p\to 0} \frac{\sin p}{p} \cdot \frac{1}{4\pi} = \underline{\frac{1}{4\pi}}$$

(4) $\displaystyle\lim_{\substack{\cancel{x\to\infty} \\ 1}} \frac{\cos\left(\frac{\pi}{2}x\right)}{x-1}$ 〔東海大〕

$x+p=1$ とおいて $p\to 0$ に近づくことを考える.

$x=1-p$ より

$$\frac{\cos\left(\frac{\pi}{2}x\right)}{x-1} = \frac{\cos\left\{\frac{\pi}{2}(1-p)\right\}}{-p} = \frac{\sin\frac{\pi}{2}p}{-p}$$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos\left(\frac{\pi}{2}-\frac{\pi}{2}p\right) = 0 + \sin\frac{\pi}{2}\cdot\sin\frac{\pi}{2}p = \sin\frac{\pi}{2}p$

$$\therefore \lim_{p\to 0} \frac{\sin\frac{\pi}{2}p}{-p} = \lim_{p\to 0} \frac{\sin\frac{\pi}{2}p}{\frac{\pi}{2}p}\cdot -\frac{\pi}{2} = \underline{-\frac{\pi}{2}}$$