



次の極限を求めよ。

- (1) $\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos x}$
- (3) $\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos 5x}$
- (5) $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x \sin 2x}$

- (2) $\lim_{x \rightarrow 0} \frac{1 - \cos 3x}{x}$
- (4) $\lim_{x \rightarrow 0} \frac{x \sin x}{1 - \cos x}$
- (6) $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$

$(\lim_{x \rightarrow 0} \frac{\sin x}{x} = \frac{\tan x}{x} = 1)$

(1) $\lim_{x \rightarrow 0} \frac{x^2(1 + \cos x)}{1 - \cos^2 x} = \lim_{x \rightarrow 0} \frac{x^2}{\sin^2 x} \cdot (1 + \cos x) = 2$

(2) $\lim_{x \rightarrow 0} \frac{1 - \cos^2 3x}{x(1 + \cos 3x)} = \lim_{x \rightarrow 0} \left(\frac{\sin 3x}{3x} \right)^2 \cdot \frac{9x}{1 + \cos 3x} = 0$

(3) $\lim_{x \rightarrow 0} \frac{1 - \cos^2 5x}{x^2(1 + \cos 5x)} = \lim_{x \rightarrow 0} \left(\frac{\sin 5x}{5x} \right)^2 \cdot \frac{25}{1 + \cos 5x} = \frac{25}{2}$

(4) $\lim_{x \rightarrow 0} \frac{x \sin x (1 + \cos x)}{1 - \cos^2 x} = \lim_{x \rightarrow 0} \frac{x}{\sin x} (1 + \cos x) = 2$

(5) $\lim_{x \rightarrow 0} \frac{1 - \cos^2 2x}{x \sin 2x (1 + \cos 2x)} = \lim_{x \rightarrow 0} \frac{\sin^2 2x}{2x} \cdot \frac{2}{1 + \cos 2x} = 1$

(6) $\lim_{x \rightarrow 0} \frac{\sin x - \sin x \cos x}{x^3 \cos x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{(1 - \cos x)}{x^2 \cos x}$
 $= \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{1 - \cos^2 x}{x^2 \cos x (1 + \cos x)}$
 $= \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{\sin^2 x}{x^2} \cdot \frac{1}{\cos x (1 + \cos x)}$
 $= \frac{1}{2}$