3C積分64

1 3[?]

次の定積分を求めよ。

(1) $\displaystyle\int_1^2 \log x\,dx$　　(2) $\displaystyle\int_0^1 \log(x^2+1)\,dx$

(3) $\displaystyle\int_0^e x\log x\,dx$　　(4) $\displaystyle\int_0^1 x^2e^x\,dx$

〔基本問題〕

(1)

$$与式 = \Big[x\log x\Big]_1^2 - \int_1^2 dx = \underline{2\log 2 - 1}$$

(2)

$$与式 = \Big[x\log(x^2+1)\Big]_0^1 - \int_0^1 \frac{2x^2}{x^2+1}\,dx$$

$$= \log 2 - 2\int_0^1\left(1-\frac{1}{x^2+1}\right)dx$$ $x=\tan\theta$ とおいて $x:0\to 1$ $\theta:0\to\frac{\pi}{4}$

$$= \log 2 - 2\Big[x\Big]_0^1 + 2\Big[\theta\Big]_0^{\frac{\pi}{4}}$$

$$= \underline{\log 2 - 2 + \frac{\pi}{2}}$$

(3)

$$与式 = \left[\frac{x^2}{2}\cdot\log x\right]_1^e - \frac{1}{2}\int_1^e x^2\cdot\frac{1}{x}\,dx$$

$$= \frac{e^2}{2} - \frac{1}{2}\left[\frac{x^2}{2}\right]_1^e = \frac{e^2}{2} - \frac{1}{4}e^2 + \frac{1}{4} = \underline{\frac{1}{4}e^2 + \frac{1}{4}}$$

(4)

$\Big[e^x\Big]_0^1$

$$与式 = \Big[x^2e^x\Big]_0^1 - \int_0^1 2xe^x\,dx = e - 2\left(\Big[xe^x\Big]_0^1 - \int_0^1 e^x\,dx\right)$$

$$= e - 2\{e-(e-1)\} = \underline{e-2}$$