

次の定積分を求めよ。

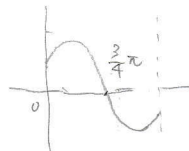
(1)  $\int_0^{\pi} |\sin x + \cos x| dx$

(2)  $\int_1^{2\pi} |2\cos^2 x + \sin x - 1| dx$

〔(2) 金沢大〕

(1)

与式 =  $\int_0^{\pi} \sqrt{2} \left| \sin\left(x + \frac{\pi}{4}\right) \right| dx$



$$= \int_0^{\frac{3\pi}{4}} \sqrt{2} \sin\left(x + \frac{\pi}{4}\right) dx - \int_{\frac{3\pi}{4}}^{\pi} \sqrt{2} \sin\left(x + \frac{\pi}{4}\right) dx$$

$$= \sqrt{2} \left[ -\cos\left(x + \frac{\pi}{4}\right) \right]_0^{\frac{3\pi}{4}} - \sqrt{2} \left[ -\cos\left(x + \frac{\pi}{4}\right) \right]_{\frac{3\pi}{4}}^{\pi}$$

$$= \sqrt{2} \left(1 + \frac{1}{\sqrt{2}}\right) - \sqrt{2} \left(\frac{1}{\sqrt{2}} - 1\right) = \sqrt{2} + 1 - 1 + \sqrt{2} = \underline{\underline{2\sqrt{2}}}$$

(2)

$$2\cos^2 x + \sin x - 1 = 2(1 - \sin^2 x) + \sin x - 1 = -2\sin^2 x + \sin x + 1$$

$$= -(\sin x - 1)(2\sin x + 1) \dots \textcircled{1} \sin x - 1 \leq 0 \text{ かつ } 2\sin x + 1 < 0 \text{ のとき}$$

$$\text{つまり } \sin x < -\frac{1}{2} \text{ のとき } \sin x < -\frac{1}{2} \text{ となる } \frac{7\pi}{6} < x < \frac{11\pi}{6}$$

$$-\frac{1}{2} \leq \sin x \leq 1 \text{ のとき } \textcircled{1} \text{ は } 0 \text{ 以上 } 1 \text{ 以下 } \textcircled{2} \text{ となる } \sin x > 1 \text{ のとき}$$

$$\begin{aligned} \therefore \text{与式} &= \int_0^{\frac{7\pi}{6}} (2\cos^2 x + \sin x - 1) dx - \int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} (2\cos^2 x + \sin x - 1) dx \\ &\quad + \int_{\frac{11\pi}{6}}^{2\pi} (2\cos^2 x + \sin x - 1) dx \dots \textcircled{2} \end{aligned}$$

$$\Rightarrow 2\cos^2 x = \cos 2x + 1 \text{ より } 2\cos^2 x + \sin x - 1 = \cos 2x + \sin x$$

f) ② は

$$\textcircled{2} = \int_0^{\frac{7\pi}{6}} (\cos 2x + \sin x) dx - \int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} (\cos 2x + \sin x) dx + \int_{\frac{11\pi}{6}}^{2\pi} (\cos 2x + \sin x) dx$$

$$= \left[ \frac{\sin 2x}{2} - \cos x \right]_0^{\frac{7\pi}{6}} - \left[ \frac{\sin 2x}{2} - \cos x \right]_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} + \left[ \frac{\sin 2x}{2} - \cos x \right]_{\frac{11\pi}{6}}^{2\pi}$$

$$= \left( \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2} \right) + 1 - \left\{ \left( -\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{2} \right) - \left( \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2} \right) \right\} + \left\{ -1 - \left( -\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{2} \right) \right\}$$

$$= 2 \left( \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2} \right) - 2 \left( -\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{2} \right) = \underline{\underline{3\sqrt{3}}}$$