次の定積分を求めよ。

(1) $\displaystyle\int_{-\pi}^{\pi} (2\sin x + \cos x)\, dx$

(2) $\displaystyle\int_{-1}^{1} (x - \tan x + 1)\, dx$

(3) $\displaystyle\int_{-\sqrt{3}}^{\sqrt{3}} \frac{x^3 + x + 1}{x^2 + 1}\, dx$

(4) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \sin^6 x\, dx$

〔基本問題〕

(1)

$$\text{与式} = 2\int_0^{\pi} \cos x\, dx = 2\Big[\sin x\Big]_0^{\pi} = 0$$

(2)

$$\text{与式} = 2\int_0^{1} dx = \Big[2x\Big]_0^{1} = 2$$

(3)

$$\text{与式} = \int_{-\sqrt{3}}^{\sqrt{3}} \left(x + \frac{1}{x^2+1}\right) dx = 2\int_0^{\sqrt{3}} \frac{1}{x^2+1}\, dx \qquad x = \tan\theta \text{とおくと}$$

$$x : 0 \to \sqrt{3} \qquad \theta : 0 \to \frac{\pi}{3} \qquad dx = (1 + \tan^2\theta)\, d\theta$$

$$= 2\int_0^{\frac{\pi}{3}} \frac{1}{\tan^2\theta + 1}(1 + \tan^2\theta)\, d\theta = 2\Big[\theta\Big]_0^{\frac{\pi}{3}} = \frac{2}{3}\pi$$

(4)

$$\text{与式} = 2\int_0^{\frac{\pi}{2}} \sin^6 x \cos x\, dx = \frac{2}{7}\Big[\sin^7 x\Big]_0^{\frac{\pi}{2}} = \frac{2}{7}$$