3C積分61

H28.7.27 訂正

次の定積分を求めよ。

(1) $\displaystyle\int_0^1 \frac{3}{(x-2)(x+1)}\,dx$　　(2) $\displaystyle\int_1^2 \frac{e^x}{e^{2x}-1}\,dx$

(3) $\displaystyle\int_0^{\frac{\pi}{6}} \frac{\cos x}{\cos 2x}\,dx$　　(4) $\displaystyle\int_2^4 \frac{1}{\sqrt{16-x^2}}\,dx$

〔基本問題〕

(1)

$$与式 = \int_0^1 \left(\frac{1}{x-2} - \frac{1}{x+1}\right) dx = \Big[\log|x-2| - \log|x+1|\Big]_0^1$$

$$= \left[\log\left|\frac{x-2}{x+1}\right|\right]_0^1 = \log\frac{1}{2} - \log 2 = \log 2^{-1} - \log 2$$

$$= -\log 2 - \log 2 = -2\log 2$$

(2)

$$与式 = \frac{1}{2}\int_1^2 \left(\frac{e^x}{e^x-1} - \frac{e^x}{e^x+1}\right) dx = \frac{1}{2}\Big[\log|e^x-1| - \log|e^x+1|\Big]_1^2$$

$$= \frac{1}{2}\left[\log\left|\frac{e^x-1}{e^x+1}\right|\right]_1^2 = \frac{1}{2}\left(\log\frac{e^2-1}{e^2+1} - \log\frac{e-1}{e+1}\right)$$

$$= \frac{1}{2}\log\frac{(e+1)^2}{e^2+1} = \underline{\log\frac{e+1}{\sqrt{e^2+1}}}\ //$$

(3)

$$与式 = \int_0^{\frac{\pi}{6}} \frac{\cos x}{1-2\sin^2 x}\,dx = \frac{1}{2}\int_0^{\frac{\pi}{6}} \left(\frac{\cos x}{1+\sqrt{2}\sin x} + \frac{\cos x}{1-\sqrt{2}\sin x}\right) dx$$

$$= \frac{1}{2\sqrt{2}}\Big[\log|1+\sqrt{2}\sin x| + \log|1-\sqrt{2}\sin x|\Big]_0^{\frac{\pi}{6}}$$

$$= \frac{1}{2\sqrt{2}}\left\{\log\left(1+\frac{\sqrt{2}}{2}\right)\left(1-\frac{\sqrt{2}}{2}\right)\right\}$$

$$= \frac{\sqrt{2}}{4}\log\left(1-\frac{1}{2}\right) = \frac{\sqrt{2}}{4}\log\frac{1}{2} = \underline{-\frac{\sqrt{2}}{4}\log 2}\ //$$

(4)

$x = 4\sin t$ とおくと　$dx = 4\cos t\,dt$　　$x: 2 \to 4$　$t: \frac{\pi}{6} \to \frac{\pi}{2}$

$$与式 = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{4\cos t}{4\sqrt{1-\sin^2 t}}\,dt = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} dt = \frac{\pi}{2} - \frac{\pi}{6} = \underline{\frac{\pi}{3}}\ //$$