

次の不定積分を求めよ。

(1) $\int_0^1 (3x+2)^4 dx$

(2) $\int_0^1 \frac{1}{x-2} dx$

(3) $\int_0^{\frac{\pi}{2}} \sin 3x \cos x dx$

(4) $\int_0^{\frac{\pi}{4}} \sin x \sin 3x dx$

(5) $\int_{-\pi}^0 \cos 2x \cos 3x dx$

(6) $\int_0^{\frac{\pi}{2}} \sin 2x(1+\cos x) dx$

〔基本問題〕

$$(1) \quad \text{与式} = \left[\frac{1}{15} (3x+2)^5 \right]_0^1 = \frac{3125}{15} - \frac{32}{15} = \frac{3093}{15} = \underline{\underline{\frac{1031}{5}}}$$

$$(2) \quad \text{与式} = \left[\log |x-2| \right]_0^1 = \log 1 - \log 2 = \underline{\underline{-\log 2}}$$

$$(3) \quad \text{与式} = \frac{1}{2} \int_0^{\frac{\pi}{2}} \sin(3x+x) + \sin(3x-x) dx = \frac{1}{2} \int_0^{\frac{\pi}{2}} (\sin 4x + \sin 2x) dx$$

$$= \frac{1}{2} \left[-\frac{1}{4} \cos 4x - \frac{1}{2} \cos 2x \right]_0^{\frac{\pi}{2}} = \frac{1}{2} \left\{ \left(-\frac{1}{4} + \frac{1}{2} \right) - \left(-\frac{1}{4} - \frac{1}{2} \right) \right\} = \underline{\underline{\frac{1}{2}}}$$

$$(4) \quad \text{与式} = -\frac{1}{2} \int_0^{\frac{\pi}{4}} (\cos 4x - \cos 2x) dx = -\frac{1}{2} \left[\frac{\sin 4x}{4} - \frac{\sin 2x}{2} \right]_0^{\frac{\pi}{4}}$$

$$= -\frac{1}{2} \left\{ \left(0 - \frac{1}{2} \right) - 0 \right\} = \underline{\underline{\frac{1}{4}}}$$

$$(5) \quad \text{与式} = \frac{1}{2} \int_{-\pi}^0 (\cos 5x + \cos x) dx = \frac{1}{2} \left[\frac{\sin 5x}{5} + \sin x \right]_{-\pi}^0 = \underline{\underline{0}}$$

$$(6) \quad \text{与式} = \int_0^{\frac{\pi}{2}} (\sin 2x + \sin 2x \cos x) dx = \int_0^{\frac{\pi}{2}} \sin 2x + \frac{1}{2} (\sin 3x + \sin x) dx$$

$$= -\frac{1}{2} \left[\cos 2x + \frac{\cos 3x}{3} + \cos x \right]_0^{\frac{\pi}{2}} = -\frac{1}{2} \left\{ (-1 + 0 + 0) - \left(1 + \frac{1}{3} + 1 \right) \right\}$$

$$= -\frac{1}{2} \cdot -\frac{10}{3}$$

$$= \underline{\underline{\frac{5}{3}}}$$