3C積分57

次の不定積分を求めよ。

(1) $\displaystyle\int_0^1 2^x\,dx$

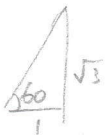
(2) $\displaystyle\int_0^{\frac{\pi}{3}} \frac{\cos 2x}{\cos^2 x}\,dx$

(3) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\sin 2x}{\cos x}\,dx$

〔基本問題〕

(1) $\left[\dfrac{2^x}{\log 2}\right]_0^1 = \dfrac{2}{\log 2} - \dfrac{1}{\log 2} = \dfrac{1}{\log 2}$

(2) $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1$ より




与式 $= \displaystyle\int_0^{\frac{\pi}{3}} \left(2 - \frac{1}{\cos^2 x}\right) dx = \left[2x - \tan x\right]_0^{\frac{\pi}{3}}$

$= \dfrac{2}{3}\pi - \sqrt{3}$

(3) $\sin 2x = 2\sin x \cos x$ より

与式 $= \displaystyle\int_0^{\frac{\pi}{4}} 2\sin x\,dx = -2\left[\cos x\right]_0^{\frac{\pi}{4}}$

$= -2\left[\dfrac{1}{\sqrt{2}} - 1\right]$

$= -\sqrt{2} + 2$