3C積分53

次の不定積分を求めよ。

(1) $\int xe^{-x}\,dx$

(2) $\int (x+1)e^{x}\,dx$

(3) $\int x^2e^{x}\,dx$

(4) $\int x\cos x\,dx$

(5) $\int e^{x}\sin x\,dx$

〔基本問題〕

(1) $\int xe^{-x}dx = -xe^{-x} + \int e^{-x}dx = -xe^{-x} - e^{-x} + C$

$\therefore$ $\underline{-e^{-x}(x+1)+C}$

(2) $\int (x+1)e^{x}dx = (x+1)e^{x} - \int e^{x}dx = (x+1)e^{x} - e^{x} + C$

$\therefore$ $\underline{xe^{x}+C}$

(3) $\int x^2e^{x}dx = x^2e^{x} - \int 2xe^{x}dx$

$= x^2e^{x} - \left(2xe^{x} - 2\int e^{x}dx\right)$

$= x^2e^{x} - 2xe^{x} + 2e^{x} + C$

$\therefore$ $\underline{e^{x}(x^2-2x+2)+C}$

(4) $\int x\cos x\,dx = x\sin x - \int \sin x\,dx$

$= x\sin x + \cos x + C$

(5) $\int e^{x}\sin x\,dx = -e^{x}\cos x + \int e^{x}\cos x\,dx$

$= -e^{x}\cos x + e^{x}\sin x - \int e^{x}\sin x\,dx$

$\therefore$ $2\int e^{x}\sin x\,dx = e^{x}(\sin x - \cos x)$

よって $\underline{\int e^{x}\sin x\,dx = \frac{e^{x}(\sin x - \cos x)}{2} + C}$