

3C積分52

次の不定積分を求めよ。

(1)  $\int \frac{e^x}{e^x+1} dx$

(2)  $\int \frac{e^{2x}}{e^x-1} dx$

(3)  $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

(4)  $\int \frac{(\log x)^3}{x} dx$

(5)  $\int \frac{\sqrt{\log x}}{x} dx$

(6)  $\int \frac{\sqrt[3]{\log x}}{x} dx$

[基本問題]

(1)  $e^x+1=t$  とおくと  $e^x dt = dt$

$\therefore$  式 =  $\int \frac{1}{t} dt = \log |t| + C$

$\log(e^x+1) + C$

(2)  $e^x=t$  とおくと  $e^x dx = dt$

$\therefore$  式 =  $\int \frac{t}{t-1} dt = \int (1 + \frac{1}{t-1}) dt = t + \log |t-1| + C$

$e^x + \log |e^x-1| + C$

(3)  $\sqrt{x}=t$  とおくと  $x=t^2$   $dx=2t dt$

式 =  $\int \frac{e^t}{t} \cdot 2t dt = \int 2e^t dt = 2e^t + C$

$2e^{\sqrt{x}} + C$

(4)  $\log x=t$  とおくと  $\frac{1}{x} dx = dt$

式 =  $\int t^3 dt = \frac{1}{4} t^4 + C$

$\frac{1}{4} (\log x)^4 + C$

(5)  $\log x=t$  とおくと  $\frac{1}{x} dx = dt$

式 =  $\int \sqrt{t} dt = \frac{2}{3} t \sqrt{t} + C$

$\frac{2}{3} \log x \sqrt{\log x} + C$

(6)  $\log x=t$  とおくと  $\frac{1}{x} dx = dt$

式 =  $\int t^{\frac{1}{3}} dt = \frac{3}{4} t^{\frac{4}{3}} + C$

$= \frac{3}{4} t^{\frac{4}{3}} + C$

$\frac{3}{4} (\log x)^{\frac{4}{3}} \sqrt[3]{\log x} + C$