

次の不定積分を求めよ。

$$(1) \int \frac{x}{1-\sqrt{x+1}} dx$$

$$(2) \int \frac{x^3}{\sqrt{x^2+1}-1} dx$$

[基本問題]

$$(1) \frac{x(1+\sqrt{x+1})}{(1-\sqrt{x+1})(1+\sqrt{x+1})} = \frac{x(1+\sqrt{x+1})}{1-x-1} = -(1+\sqrt{x+1})$$

$$\begin{aligned} \therefore \int &= -\int (1+\sqrt{x+1}) dx = -\int \left\{ 1+(x+1)^{\frac{1}{2}} \right\} dx \\ &= -x - \frac{2}{3} (x+1)^{\frac{3}{2}} + C \\ &= \underline{-x - \frac{2}{3} (x+1)\sqrt{x+1} + C} \end{aligned}$$

$$(2) \frac{x^3(\sqrt{x^2+1}+1)}{(\sqrt{x^2+1}-1)(\sqrt{x^2+1}+1)} = x(\sqrt{x^2+1}+1)$$

$$\begin{aligned} \therefore \int &= \int x\sqrt{x^2+1} dx + \int x dx \\ &= \int x(x^2+1)^{\frac{1}{2}} dx + \int x dx \\ &= \frac{1}{3} (x^2+1)^{\frac{3}{2}} + \frac{1}{2} x^2 + C \end{aligned}$$

$\frac{2}{3}$

$$\therefore \underline{\frac{1}{3} (x^2+1)\sqrt{x^2+1} + \frac{1}{2} x^2 + C}$$