

1p = 10k8

ok

$\sin \theta + \cos \theta = \frac{1}{2}$ のとき, $\sin \theta \cos \theta = \square$ であり, $\tan^2 \theta + \frac{1}{\tan^2 \theta} = \square$ 。
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$$(\sin \theta + \cos \theta)^2 = \left(\frac{1}{2}\right)^2$$

$$1 + 2\sin \theta \cos \theta = \frac{1}{4}$$

$$2\sin \theta \cos \theta = -\frac{3}{4}$$

$$\sin \theta \cos \theta = -\frac{3}{8}$$

$$\tan^2 \theta + \frac{1}{\tan^2 \theta} = \frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta}$$

$$= \frac{\sin^4 \theta + \cos^4 \theta}{\sin^2 \theta \cos^2 \theta}$$

$$= \frac{(\sin^2 \theta + \cos^2 \theta)^2 - 2\sin^2 \theta \cos^2 \theta}{\sin^2 \theta \cos^2 \theta}$$

$$= \frac{(\sin^2 \theta + \cos^2 \theta)^2 - 2(\sin \theta \cos \theta)^2}{(\sin \theta \cos \theta)^2}$$

$$= \frac{1 - 2 \cdot \left(-\frac{3}{8}\right)^2}{\left(-\frac{3}{8}\right)^2}$$

$$= \frac{64 - 18}{9}$$

$$= \frac{46}{9}$$

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$$\therefore \tan^2 \theta + \frac{1}{\tan^2 \theta} = \frac{46}{9}$$